Assignment 5.3 - RIEMANN-SUMS Due 04/12/2021

1. $\sum_{k=4}^{7} k^2 = \frac{7(7+4)(2\times7+4)}{6}$

$\frac{(77)(18)}{6} = \frac{(\cancel{56})(\cancel{15})}{6}$

$= 231$ $\quad = \cancel{140}$

2. $\sum_{i=3}^{7} (i^2 - i) =$

$\frac{7(7+3)(2\times7+3)}{6} - \frac{7(7+3)}{2}$

$\frac{(70)(17)}{6} - \frac{70}{2} =$

$198 - 35 = 163$

163

3. $\sum_{k=1}^{80} k = \frac{80(80+1)}{2}$

$= \frac{6,480}{2}$

$= 3240$

4. $\sum_{k=1}^{100} 20 = 20 \times 100$

$= 2000$

5. $\int_0^4 x^2\,dx = \lim_{n\to\infty} \sum_{k=1}^{5} \left(1 + \frac{4-0}{n}k\right)^2 \left(\frac{4-0}{n}\right)$

$= \lim_{n\to\infty} \sum_{k=1}^{5} \left(1 + \frac{4}{5}k\right)^2 \left(\frac{4}{5}\right)$

or $\Delta x = \frac{b-a}{n} = \frac{4-0}{5}$

$= 4/5$

6. $\int_0^4 x^2\,dx = \Delta x \frac{b-a}{n}$

$= \frac{4-0}{5}$

$= 4/5$

c) $\int_0^3 x^2 + 3\,dx = \lim_{n\to\infty}\left[\sum_{k=1}^{n} f(x_k^*)\Delta x\right]$

$f(x_k^*)\Delta x = \left(\frac{3i}{n} + 1\right)\frac{3}{n}$

$= \frac{9i}{n^2} + \frac{3}{n}$

d) $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x = \left(\sum_{i=1}^{n} \frac{9i}{n^2} + \sum_{i=1}^{n} \frac{3}{n}\right)$

e) $\lim_{n\to\infty} \left(\frac{9}{n^2}\sum_{i=1}^{n} i + \frac{3}{n}\sum_{i=1}^{n} 1\right]$

$\frac{9}{n^2}\left(\frac{n(n+1)}{2} + \frac{3}{n}n\right)$

$\frac{9}{2} + \frac{3+6+9}{2} + 3 = \frac{13}{2}$

$= 6.5$

7] $\int_0^4 x^2\,dx$ $\Delta x = \frac{b-a}{n}$

$= \frac{4-0}{6} = \frac{4}{6}$

$\Delta x = \frac{2}{3}$

8. $\int_1^4 5x-3\,dx = \frac{4-1}{5}$

$= \frac{3}{5}$

9] $\int_1^6 (2x-5)\,dx = \frac{6-1}{5}$

$\Delta x = \frac{5}{5} = 1$

$= 1$

10] $\int_1^5 (6x-1)\,dx = \frac{5-1}{5}$

$= \frac{4}{5}$

11] $\int_0^3 x^2+3\,dx$

(a) $= (3)(3) + \frac{1}{2}(3)(3)$

$9 + 4.5$

$= 13.5$

(b) $x_k^* = \frac{3i}{n}$

(12) $\int_0^2 2x^3\,dx$

(a) $(2)(2) + \frac{1}{2}(2)(2)$

$4 + 2 = 6$

(b) $f(x_k^*)\,\Delta x$

$\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$\left(\frac{2i}{n}+1\right)\frac{2}{n} = \left(\frac{4i}{n^2}+\frac{2}{n}\right)$

$= \left(\frac{4i}{n^2}+\frac{2}{n}\right)$

(c) $\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{4i}{n^2}+\sum_{i=1}^{n}\frac{2}{n}\right)$

$= \lim_{n\to\infty}\left(\frac{4}{n^2}\sum_{i=1}^{n} i + \frac{2}{n}\sum_{i=1}^{n} 1\right)$

(d) $= \lim_{n\to\infty}\left(\frac{4}{n^2}\frac{n(n+1)}{2} + \frac{2}{n}n\right)$

(e) $= \lim_{n\to\infty}\left(2+\frac{2}{n}+2\right)$

$= 4$